

Article

Development of separation sharpness model for hydrocyclone

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ABSTRACT

Hydrocyclones are mechanical devices used in classifying and separating many different types of materials. A classification function of the hydrocyclone has been continually developed for solid–liquid separation. In the classification process of solids from liquids, it is desirable to reduce the amount of misplaced material; therefore, the separation sharpness, α (alpha), is a parameter that helps in evaluating misplaced material and has been developed as a model to help the designer predict the performance of the classification. However, the problem with the separation sharpness model is that it cannot be used outside the range of conditions under which it was developed. Therefore, this research aimed to develop the separation sharpness model to predict more accurately and cover a wide range of conditions using the multiple linear regression method. The new regression model of separation sharpness was based on a wide range of both experimental and industrial data-sets of 431 tests collaborating with the additional experiments of 117 tests that were obtained from a total of 548 tests. The new model of separation sharpness can be used in the range of 30–762 mm hydrocyclone body diameters and feed solid concentrations in the range of 0.5wt%–80 wt%. When compared with the experimental separation sharpness, the accuracy of the separation sharpness model prediction has an error of 4.53% and R^2 of 0.973.

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1. Introduction

Hydrocyclones are widely employed in the mining and chemical industries for classification, thickening, and separation of either solids or droplets based on their size and density [1] because their advantages has a simple construction with no moving parts, a flexible capacity, low manufacturing and maintenance costs [2–5].

The study of variables affecting the separation sharpness of the hydrocyclone led to the development of the separation sharpness model of hydrocyclone to predict more accurately and to cover a wide range of applications. Separation sharpness, α (alpha), represents an amount of misplaced material in products. The general slope of the grade efficiency curve [6], is defined as the ratio of two particle sizes corresponding to two different percentages of the grade efficiency curve [7] (for example: $\alpha_{40/60} = d_{40}/d_{60}$). It had been a long-standing challenge for designers to understand the separation sharpness of the

hydrocyclone. The authors illustrated the history of separation sharpness model as shown in Fig. 1.

Fahlstorm [8] experimentally studied the influence of the viscosity of slurry and liquid, density of slurry and solids on the separation sharpness, whereas Lynch [9] conducted experiments to study the effect of the feed solid concentration on separation sharpness. Four years later, Lynch and colleagues from the Julius Kruttschnitt Mineral Research Centre (JKMRC) [10] proposed the Whiten function for a reduced efficiency curve as shown in Eq. (1). This equation was adopted to calculate the reduced efficiency curves because it represents the classification well and because the sum of the squared deviations between the experiment and the predicted values in the corrected efficiency, $E_c(d/d_{50c})$, is lower than the equation proposed by Plitt, as shown in Eq. (2). In addition, this curve is invariant with the vortex finder and spigot diameter, including the feed density and pressure [11].

In 1971, Plitt [12], at the University of Alberta, proposed the Plitt–Reid function for reduced efficiency curve (Eq. (2)), and he claimed that his function demonstrated ease of application and simplicity. Nevertheless, the applicability was quite limited by the functional form of the model [13]. At a later time, Plitt [14] initially proposed the

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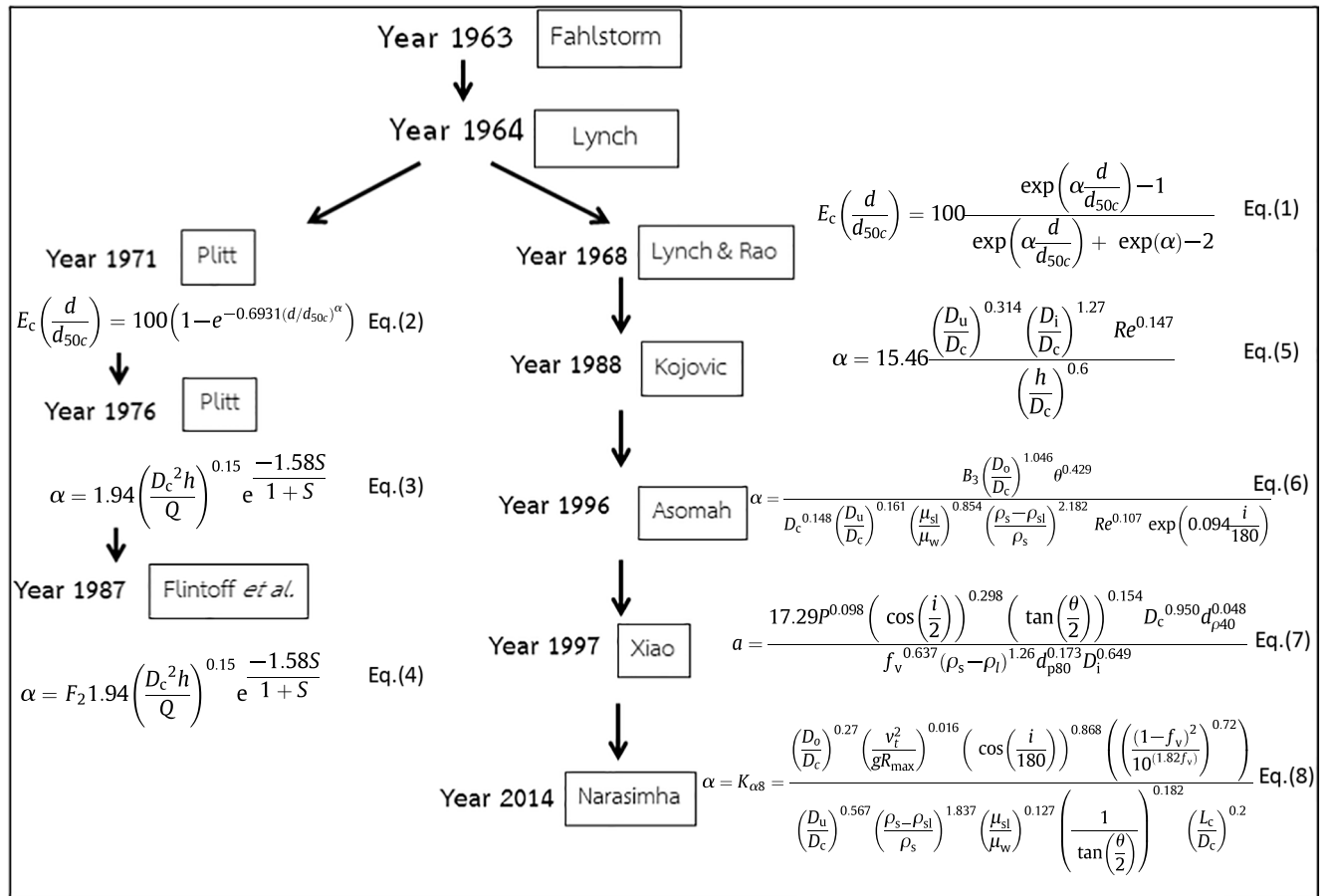


Fig. 1. History of separation sharpness model.

separation sharpness model, α , including a comprehensive model for all the performance characteristics of the hydrocyclone which were developed using multiple linear regression. In addition, he implicitly assumed the separation sharpness model to be independent of the material type in the feed, as can be seen from Eq. (3). His model was widely used to predict hydrocyclone performance. However, his model prediction was found to be inaccurate by some researchers [15]. After that, Flintoff *et al.* [16] advised incorporating the calibration parameter, F_2 , into the model, as shown in Eq. (4). They claimed that incorporating F_2 in the separation sharpness model would make it a more reliable prediction.

$$E_c\left(\frac{d}{d_{50c}}\right) = 100 \frac{\exp\left(\alpha \frac{d}{d_{50c}}\right) - 1}{\exp\left(\alpha \frac{d}{d_{50c}}\right) + \exp(\alpha) - 2} \quad (1)$$

$$E_c\left(\frac{d}{d_{50c}}\right) = 100 \left(1 - e^{-0.6931(d/d_{50c})^\alpha}\right) \quad (2)$$

$$\alpha = 1.94 \left(\frac{D_c^2 h}{Q}\right)^{0.15} \frac{-1.58S}{e^{1+S}} \quad (3)$$

$$\alpha = F_2 1.94 \left(\frac{D_c^2 h}{Q}\right)^{0.15} \frac{-1.58S}{e^{1+S}} \quad (4)$$

$$\alpha = 15.46 \frac{\left(\frac{D_u}{D_c}\right)^{0.314} \left(\frac{D_i}{D_c}\right)^{1.27} Re^{0.147}}{\left(\frac{h}{D_c}\right)^{0.6}} \quad (5)$$

$$\alpha = \frac{B_3 \left(\frac{D_o}{D_c}\right)^{1.046} \theta^{0.429}}{D_c^{0.148} \left(\frac{D_u}{D_c}\right)^{0.161} \left(\frac{\mu_{sl}}{\mu_w}\right)^{0.854} \left(\frac{\rho_s - \rho_{sl}}{\rho_s}\right)^{2.182} Re^{0.107} \exp\left(0.094 \frac{i}{180}\right)} \quad (6)$$

$$\alpha = \frac{17.29 p^{0.098} \left(\cos\left(\frac{i}{2}\right)\right)^{0.298} \left(\tan\left(\frac{\theta}{2}\right)\right)^{0.154} D_c^{0.950} d_{p40}^{0.048}}{f_v^{0.637} (\rho_s - \rho_l)^{1.26} d_{p80}^{0.173} D_i^{0.649}} \quad (7)$$

$$\alpha = K_{\alpha 8} = \frac{\left(\frac{D_o}{D_c}\right)^{0.27} \left(\frac{v_t^2}{g R_{max}}\right)^{0.016} \left(\cos\left(\frac{i}{180}\right)\right)^{0.868} \left(\frac{(1-f_v)^2}{10^{(1.82f_v)}}\right)^{0.72}}{\left(\frac{D_u}{D_c}\right)^{0.567} \left(\frac{\rho_s - \rho_{sl}}{\rho_s}\right)^{1.837} \left(\frac{\mu_{sl}}{\mu_w}\right)^{0.127} \left(\frac{1}{\tan\left(\frac{\theta}{2}\right)}\right)^{0.182} \left(\frac{L_c}{D_c}\right)^{0.2}} \quad (8)$$

The development of a separation sharpness model has been ongoing. Kojovic [17] developed the regression model of separation sharpness as shown in Eq. (5), using the experimental data of 48 tests from Nageswararao [18] and 116 tests from Rao [11]. Data-sets of 164 tests with a feed solid concentration (14wt%–65 wt%) including silica and limestone were used as the feed material.

The prediction accuracy of the Kojovic model had a percentage error (error), $\varepsilon_{Kojovic}$, of approximately 19%. Comparing with the Plitt model (Eq. (3)), it was found that the prediction accuracy of the Plitt model had a error, ε_{Plitt} , of approximately 27%. From Eq. (5), the separation sharpness was dependent not only on the operating variable, but also on the hydrocyclone dimensions.

Recently, the new empirical model of the separation sharpness was created using experimental data from 123 data-sets with a feed solid

concentration (13wt%–72 wt%) including limestone, mount Leyshon-gold ore (Australia), Pb-Zn ores, Cu-ore, and 31 used as the feed materials by Asomah [19], as shown in Eq. (6). The hydrocyclone inclination, viscosity, and density were considered, which differed from the Kojovic model (Eq. 5). Fig. 2 shows a comparison of the experimental separation sharpness, α_{exp} , and the predicted separation sharpness, α_{pred} , of the Asomah model. It revealed that the prediction of the Asomah model had less accuracy (error, $\varepsilon_{\text{Asomah}}$, of approximately 18.7%) for the values of $\alpha_{\text{exp}} > 4.5$ [20].

Xiao [21] also developed a separation sharpness model from a total of 369 data-sets, which consisted of tests undertaken by Rao [11], Nageswararao [18], Asomah [19], and some commercial tests from JKTECH and JKMRC with a feed solid concentration (13wt%–72 wt%), including silica, limestone, mount Leyshon-gold ore (Australia), Pb-Zn ores, and Cu-ore used as the feed materials. The Xiao model was defined in Eq. (7). When considering the prediction accuracy of the Xiao model, as shown in Fig. 3, it was found that the prediction of the Xiao model gave relatively poor correlations with the hydrocyclone dimension and operating conditions (error, $\varepsilon_{\text{Xiao}}$ of 66.6%). In addition, α_{pred} was sensitive to any changes in the separation size, water split, and experimental data.

Apart from these researchers, Narasimha *et al.* [22] and Narasimha [23] developed a semi-mechanistic model using a computational fluid dynamics (CFD) technique coordinating with the multiple linear regression approach and conducted an additional experiment using a 254 mm Krebs hydrocyclone in the JKMRC pilot plant, collaborating with the experimental data of other researchers with a total of 479 data-sets, as shown in Eq. (8). The additional experiments consisted of feed solid concentrations varying from 9wt% to 30 wt%, and limestone was used as the feed material.

The CFD approach was applied to improve the separation sharpness model, which can predict more accurately and covered a wide range of operating conditions of the hydrocyclone. When comparing the Narasimha model with the Asomah model, the result indicated that the error of the Narasimha model, $\varepsilon_{\text{Narasimha}}$, was 23.4% whereas $\varepsilon_{\text{Asomah}}$ was 41.5% as shown in Fig. 4.

As reviewed in the literature above, several data gaps have been revealed. The accuracy of the separation sharpness model prediction was in the range of 18.7%–66.6% error and can be used in the range of 102–500 mm hydrocyclone body diameters with feed solid concentrations in

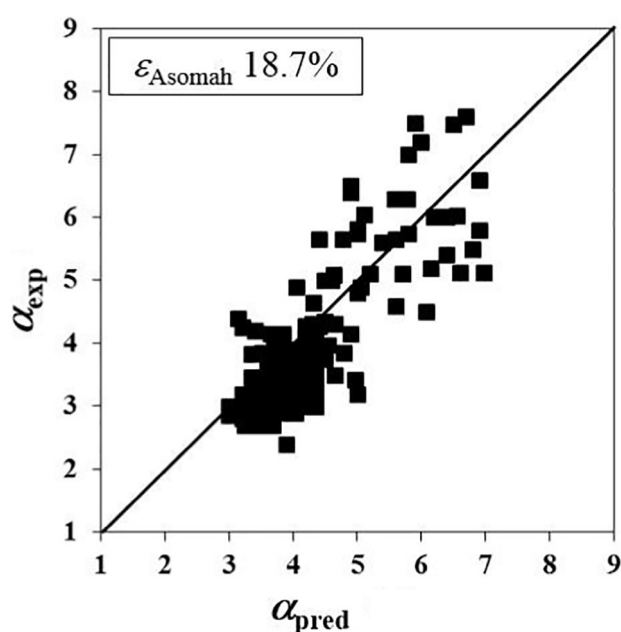


Fig. 2. Comparison of α_{exp} and α_{pred} of the Asomah model (1996).

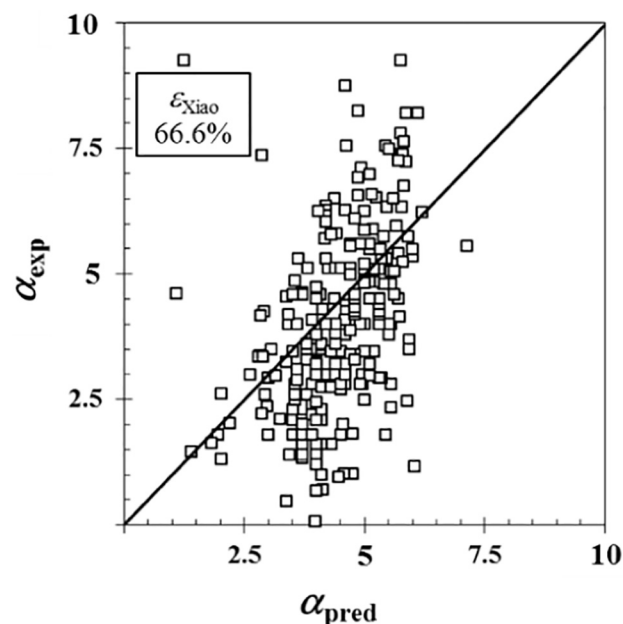


Fig. 3. Comparison of α_{exp} and α_{pred} of the Xiao model (1997).

the range of 9wt%–70 wt%, and the limitation of the feed material type used. Therefore, additional experiments were conducted to fill the data gaps (e.g., the small hydrocyclone body diameter, low feed solid concentration, and various types of feed material).

To predict more accurately and cover a wide range of conditions, the aim of this research was to develop the empirical model of the separation sharpness for hydrocyclones using a multiple linear regression and dimensionless approach.

2. Data-Sets for Developing the Separation Sharpness Model

The development of the separation sharpness model was based on historical data-sets (*i.e.* those of Rao [30], Nageswararao [18], Hinde [24], Castro [25], Asomah [19], Mainza [26], Narasimha [23]) and additional experiments (*i.e.* those of the Nanotec hydrocyclone research team), as shown in Table 1, for a total of 548 tests by using the geometry and dimension of hydrocyclone as shown in Fig. 5. A large number of entire data-sets were obtained from various hydrocyclone body diameters and types of materials based on various aims of the tests. The

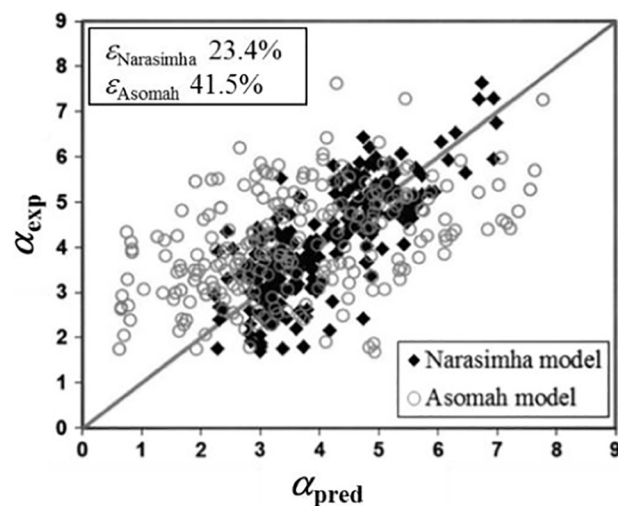


Fig. 4. Comparison of the Narasimha model (2014) and the Asomah model (1996).

Table 1

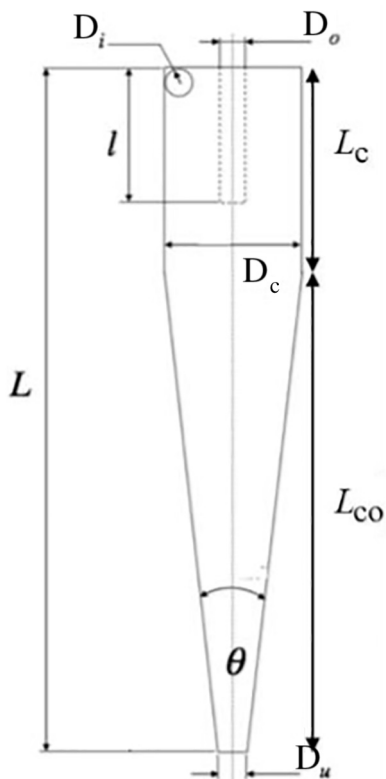
Entire data-sets used for model development

Data-sets	Hydrocyclone body diameters /mm	Number of tests	Type of feed materials	Aim of the tests
Rao (1966)	152	26	Silica	Dimension variables effect
	508	92		
	152	24		Cone angle and hydrocyclone length effects
Nageswararao (1978)	254	11	Limestone	Dimension variables effect
	381	14		
	760	40		Dimension and operating variables effect
Hinde (1985)	254	30	Limestone	Viscosity effect
Castro (1990)	508	26	Mt. Isa-copper ore	
		17	Mineral identity tests	
Asomah (1996)	102	17	Limestone	Density effect
		21	Mt. Leyshon-gold ore	Inclination effect (0°–67.5°)
		10	Pb–Zn ores	Inclination effect (0°–135°)
		32	Cu-ore	Inclination effect (0°–90°)
		40	UG2 platinum ore	Inclination effect (0°–135°)
Mainza (2006)	600	40	UG2 platinum ore	Dimension variables effect
Narasimha (2009)	254	31	Limestone	Cone angle and feed solid concentration effects
Nanotec hydrocyclone research team	30	18	Soil	Feed-flow rate, spigot diameter and feed solid concentration effects
	40	27	Silica	Vortex finder, cylindrical and conical lengths effects
	50	30	Soil	Feed-flow rate, spigot diameter and feed solid concentration effects
	100	26	Soil	Feed-flow rate, spigot diameter and feed solid concentration effects
	125	16	Soil and bagasse	Feed-flow rate and spigot diameter effects
Total data-sets		548		

composition of the data-sets was collected from a series of tests conducted with experimental and industrial hydrocyclone body diameters.

Details of these experimental conditions that previous researchers used to create the separation sharpness models were used in this research to develop the new separation sharpness model as follows.

Rao [11] experimentally studied the effects of change in the dimensions of the vortex finder and spigot, feed pressure and feed solid concentrations (14wt%–65 wt%) on capacities and classification performance using the Krebs hydrocyclone body diameters of 152.4 and 508 mm, including silica as the feed material.

**Fig. 5.** Geometry and dimension of hydrocyclone.

In 1978, Nageswararao used the experimental data from Lynch and Rao [27] and Rao [11] and conducted additional experiments using the 152 mm Krebs hydrocyclone body diameter to study the effects of both the cone angle and hydrocyclone length on the hydrocyclone performance, whereas the 254 and 381 mm hydrocyclone body diameters were used to investigate the effect of dimension variables (*i.e.* hydrocyclone body diameter and inlet diameter) on hydrocyclone performance with limestone as a feed material and using feed solid concentrations (50wt%–65 wt%).

Hinde *et al.* [28] and Mackay *et al.* [29] performed tests using the 760 mm hydrocyclone body diameter with both spray and roped conditions of the underflow including feed solid concentrations (50wt%–70 wt%). Gold minerals were used as the feed material.

The aim of the test work was to examine the effect of the inlet, vortex finder, and spigot diameters and feed-flow rate and volumetric fraction of solids in the feed on the classification of hydrocyclone performance.

Later, Castro [25] showed experimental results of the effect of viscosity, spigot and feed solid concentrations (20wt%–78 wt%) on the separation efficiency of the hydrocyclone using 254 and 762 mm hydrocyclone body diameters with limestone used as feed material, whereas Mount Isa copper ore (Australia) was used as the feed material with a 508 mm hydrocyclone body diameter to determine the effect of viscosity on the hydrocyclone performance.

Asomah [19] experimentally studied the effect of different hydrocyclone inclinations and densities on the separation efficiency of the hydrocyclone using hydrocyclone body diameters of 102 and 508 mm with feed solid concentrations (13wt%–72 wt%) and using limestone, Mount Leyshon-gold ore (Australia), Pb–Zn ores, Cu-ore as feed materials.

After that, Mainza [30] investigated the influence of the inner and outer vortex finder diameter on the hydrocyclone (three-product hydrocyclone) performance using the 600 mm hydrocyclone body diameter and feed solid concentrations (40wt%–60 wt%). The UG2 platinum ore was employed as the feed material in this test. In addition to these researchers, Narasimha [23] experimentally studied the effect of cone angles (10.5° and 20°) and feed solid concentrations (9wt%–30 wt%) on the hydrocyclone efficiency using a 254 mm Krebs hydrocyclone in the JKMR pilot plant and limestone as feed material.

The additional experiments from Nanotec hydrocyclone research team were conducted using 30, 40, 50, 100, and 125 mm hydrocyclone body diameters following the hydrocyclone proportion of Wongsarivej [31] with solid feed concentrations in the range of 0.5wt%–3 wt%, including silica, soil, and bagasse used as feed materials.

As described, the entire data-sets used for model development above showed that the new separation sharpness model can be used in the range of 30–760 mm hydrocyclone body diameters, with 0.5wt%–80 wt % feed solid concentrations and with these feed material types: silica, limestone, gold minerals, Mt. Isa-copper ore, Mt. Leyshon-gold ore, Pb-Zn ores, Cu-ore, UG2 platinum ore, soil, and bagasse.

3. Separation Sharpness Model Development

The dimensionless approach was used to formulate a new separation sharpness model because of the advantage of creating the dimensional consistency for scaling up. A number of dimensionless groups, which consist of dimension, feed material, medium and operating variables, were chosen as the components for developing the form of the separation sharpness model in predicting the separation sharpness more accurately. These dimensionless groups were described as follows:

- 1) In terms of dimension variables, the hydrocyclone body diameter, D_c , was chosen as the characteristic dimension of length, and for all other physical hydrocyclone dimensions were normalized. The dimensionless variables were D_i/D_c , D_o/D_c , D_u/D_c , l/D_c , L_c/D_c , L_{co}/D_c . In addition, Neesse [32] and Neesse *et al.* [33] clearly described the turbulent mixing, which became less important with larger particles and significantly affected the diffusion of very fine particles. This turbulent diffusion coefficient, D_t , can be defined as shown in Eq. (9):

$$\frac{D_t}{k_r D_c} = \tan^{(\theta/2)} \quad (9)$$

where k_r is the radial velocity constant.

- 2) The terms of the feed material and medium variables were the relative viscosity, μ_{sl}/μ_l , hindered settling factor, v_h/v_t , and relative density, $(\rho_s - \rho_{sl})/\rho_s$. Initially, the dimensionless group, relative viscosity, as suggested by Ishii and Mishima (1984), is defined in Eq. (10), which is applied to all general slurry. Nevertheless, this model did not consider the effect of the fine fraction (below 38 μm) on slurry viscosity and assumed the slurry to exhibit Newtonian behavior [34]. Therefore, to cover this effect, the predicted slurry viscosity model was developed as shown in Eq. (11), and it was calibrated against the experimental viscosity data [19,25].

$$\frac{\mu_{sl}}{\mu_l} = \frac{1}{\left(1 - \frac{f_v}{0.62}\right)^{1.55}} \quad (10)$$

$$\frac{\mu_{sl}}{\mu_l} = \frac{(F_{-38\mu})^{0.39}}{\left(1 - \frac{f_v}{0.62}\right)^{1.55}} \quad (11)$$

where $F_{-38\mu}$ is the fine fraction below 38 μm in hydrocyclone feed. When considering the dimensionless group of the hindered settling factor, Steinnour [35] defined the relation between the volume fraction of the solid in the feed, f_v , and the settling rate, v_h/v_t , as shown in Eq. (12). In other words, the solid concentration and size distribution in the feed strongly affected hydrocyclone separation by blocking the movement of individual particles, resulting in the change of the particle settling rate [18,22].

$$\frac{v_h}{v_t} = \frac{(1 - f_v)^2}{10^{(1.82f_v)}} \quad (12)$$

- 3) The terms of operating variables were P -number, γ , Reynolds number, Re , and hydrocyclone inclination, $\cos(i/2)$. In general, the flow

of fluid inside the hydrocyclone acts differently according to the velocity of flow and loss of energy in the flow. The dimensionless groups used to describe these mechanisms are the Euler number, Eu , and Froude number, Fr , consisting of a new term called the P -number, which is the ratio of pressure drop in the flow to gravitational forces [21]. The P -number was defined as shown in Eq. (13):

$$\gamma = EuFr = \left(\frac{P}{\rho_{sl} v^2}\right) \left(\frac{v^2}{g D_c}\right) = \frac{P}{\rho_{sl} g D_c} \quad (13)$$

In addition to the P -number, the other terms of operating variables are the Reynolds number, Re , and hydrocyclone inclination, $\cos(i/2)$. Heiskanen [36] proposed that describing the amount of turbulence in particle settling used Re , which is the ratio of the inertia force to the viscous force of fluid, as shown in the following equation:

$$Re = \frac{\text{Inertia Force}}{\text{Viscous Force}} = \frac{(\rho_s - \rho_{sl}) v_i D_c}{\mu} \quad (14)$$

where

$$v_i = \frac{Q_f}{A_i} \quad (15)$$

The multiple linear regression technique was used to analyze the relationship between dependent variables and independent variables, leading to creating a regression equation to predict the experimental data more accurately and to cover a wide range of conditions. This multiple linear regression equation is presented as follows:

$$Y = a_0 + n_1 X_1 + n_2 X_2 + \dots + n_k X_k \quad (16)$$

An increase in the number of independent variables usually results in a higher coefficient of determination, R^2 , and a lower error, ε . The R^2 is a key output of regression analysis. It is interpreted as the proportion of the variance in the dependent variable that is predictable based on the independent variables, as shown in Eq. (17). The error calculation of the model can be defined as shown in the following expression [21,23]:

$$R^2 = \frac{\sum (\hat{y} - \bar{y})^2}{\sum (y - \bar{y})^2} \quad (17)$$

$$\varepsilon = 100 \sqrt{\frac{\sum_{i=1}^N \left(\frac{\alpha_{\text{exp}_i} - \alpha_{\text{pred}_i}}{\alpha_{\text{exp}_i}} \right)^2}{N}} \quad (18)$$

To develop the separation sharpness model in the form of an exponent equation, as shown in the following Eq. (19), this equation must be transformed to the form of a multiple linear regression equation, Eq. (20), by taking the natural logarithm:

$$Y = k_1 X_1^{n_1} X_2^{n_2} \dots X_k^{n_k} \quad (19)$$

$$\ln(Y) = \ln(k_1) + n_1 \ln(X_1) + n_2 \ln(X_2) + \dots + n_k \ln(X_k) \quad (20)$$

The value of the constant k_1 ($\alpha_0 = \ln k_1$) and regression coefficients $n_1, \dots, n_k$ in Eq. (20) can be determined by solving Eq. (21) and comparing the coefficients. The constant and regression

coefficients are then substituted into Eq. (19) to achieve the exponent equation:

$$\begin{bmatrix} \sum_{i=1}^n Y_i \\ \sum_{i=1}^n X_{1i} Y_i \\ \sum_{i=1}^n X_{2i} Y_i \\ \vdots \\ \sum_{i=1}^n X_{ki} Y_i \end{bmatrix} = \begin{bmatrix} z & \sum_{i=1}^n X_{1i} & \sum_{i=1}^n X_{2i} & \cdots & \sum_{i=1}^n X_{ki} \\ \sum_{i=1}^n X_{1i} & \sum_{i=1}^n X_{1i} X_{1i} & \sum_{i=1}^n X_{1i} X_{2i} & \cdots & \sum_{i=1}^n X_{1i} X_{ki} \\ \sum_{i=1}^n X_{2i} & \sum_{i=1}^n X_{2i} X_{1i} & \sum_{i=1}^n X_{2i} X_{2i} & \cdots & \sum_{i=1}^n X_{2i} X_{ki} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \sum_{i=1}^n X_{ki} & \sum_{i=1}^n X_{ki} X_{1i} & \sum_{i=1}^n X_{ki} X_{2i} & \cdots & \sum_{i=1}^n X_{ki} X_{ki} \end{bmatrix} \times \begin{bmatrix} a_0 \\ n_1 \\ n_2 \\ \vdots \\ n_k \end{bmatrix} \quad (21)$$

The following relationship of Eq. (22) was obtained by considering all empirical hydrocyclone models of separation sharpness developed by our research team and other researchers with all test results:

$$\alpha = f \left(\frac{D_i}{D_c}, \frac{D_o}{D_c}, \frac{D_u}{D_c}, \gamma, Re, \frac{l}{D_c}, \frac{L_c}{D_c}, \frac{L_{co}}{D_c}, \cos \frac{i}{2}, \frac{(1-f_v)^2}{10^{(1.82f_v)}}, \frac{\rho_s - \rho_{sl}}{\rho_s}, \frac{\mu_{sl}}{\mu_l}, \tan \frac{\theta}{2} \right) \quad (22)$$

This relationship of variables was studied using the Excel solver program (i.e. multiple linear regression). The solver estimates each of the variable terms in the relationship of separation sharpness. Later, Eq. (22) can be transformed into the form of Eq. (23) as follows:

$$\alpha = A \left(\frac{D_i}{D_c} \right)^{n_1} \left(\frac{D_o}{D_c} \right)^{n_2} \left(\frac{D_u}{D_c} \right)^{n_3} (\gamma)^{n_4} Re^{n_5} \left(\frac{l}{D_c} \right)^{n_6} \left(\frac{L_c}{D_c} \right)^{n_7} \left(\frac{L_{co}}{D_c} \right)^{n_8} \left(\cos \frac{i}{2} \right)^{n_9} \left(\frac{(1-f_v)^2}{10^{(1.82f_v)}} \right)^{n_{10}} \left(\frac{\rho_s - \rho_{sl}}{\rho_s} \right)^{n_{11}} \left(\frac{\mu_{sl}}{\mu_l} \right)^{n_{12}} \left(\tan \frac{\theta}{2} \right)^{n_{13}} \quad (23)$$

As a result of the estimated solver, the goodness of fit, fitting statistics, and improvement over existing variables were found in the separation sharpness model. The value of the system constant, A , in Eq. (23) for the separation sharpness model was refitted for the obtained type of materials and hydrocyclone body diameters with the entire historical and additional experiment data-sets (a total of 548 tests). The developed model covers only in the range of used data. If the model is used in the prediction out of the range of used data, the prediction accuracy will probably have some errors. However, if the model is used to predict the separation sharpness of a process with different feed materials that are not used in developing model, the density of feed materials used in the model that close to the density of different feed materials may be used instead.

From Eq. (21), all the regression coefficients were $n_1 = -0.284$, $n_2 = 0.135$, $n_3 = -0.158$, $n_4 = 0.068$, $n_5 = -0.140$, $n_6 = -0.682$, $n_7 = -0.173$, $n_8 = 0.242$, $n_9 = 0.182$, $n_{10} = 0.346$, $n_{11} = -0.088$, $n_{12} = -0.190$ and $n_{13} = 0.329$. When replacing these regression coefficients in Eq. (20), the new model, which predicted α_{exp} was defined as shown in Eq. (24):

$$\alpha = A \left(\frac{D_i}{D_c} \right)^{0.135} \gamma^{0.068} \left(\frac{L_{co}}{D_c} \right)^{0.242} \left(\cos \frac{i}{2} \right)^{0.182} \left(\frac{(1-f_v)^2}{10^{(1.82f_v)}} \right)^{0.346} \left(\tan \frac{\theta}{2} \right)^{0.329} \left(\frac{D_o}{D_c} \right)^{-0.284} \left(\frac{D_u}{D_c} \right)^{-0.158} Re^{0.140} \left(\frac{l}{D_c} \right)^{-0.682} \left(\frac{L_c}{D_c} \right)^{-0.173} \left(\frac{\rho_s - \rho_{sl}}{\rho_s} \right)^{-0.088} \left(\frac{\mu_{sl}}{\mu_l} \right)^{-0.190} \quad (24)$$

According to Eq. (24), the influence of the dimension, feed material, medium, and operating variables on the separation sharpness depends on the value of the exponent of the vortex finder diameter, D_o/D_c , P -

number, γ , conical length, L_{co}/D_c , hydrocyclone inclination, $\cos(i/2)$, hindered settling factor, $(1-f_v)^2/10^{(1.82f_v)}$, relative particle density, $(\rho_s - \rho_{sl})/\rho_s$, relative slurry viscosity, μ_{sl}/μ_l , cone angle, $\tan(\theta/2)$, feed inlet diameter, D_i/D_c , spigot diameter, D_u/D_c , Reynolds number, Re , vortex finder length, l/D_c , and cylindrical length, L_c/D_c .

The exponent of l/D_c , $(1-f_v)^2/10^{(1.82f_v)}$, $\tan(\theta/2)$, D_i/D_c , L_{co}/D_c , μ_{sl}/μ_l , $\cos(i/2)$, L_c/D_c , D_u/D_c , Re , D_o/D_c , had higher values than $(\rho_s - \rho_{sl})/\rho_s$ and γ . Fig. 6 shows a comparison between the separation sharpness obtained from the experiment and prediction. Moreover, α_{pred} was in close agreement with α_{exp} . In addition, the plot between α_{exp} and α_{pred} resulted in a $\varepsilon_{New model}$ of 4.53% and $R^2_{New model}$ of 0.973, which indicates that the new model has a high reliability in forecasting the separation sharpness of the hydrocyclone.

4. Comparison of the New Model from this Research and the Narasimha Model

According to Fig. 4, the experimental data (479 tests) used by the Narasimha model (Eq. 8) resulted in a $\varepsilon_{Narasimha}$ of 23.4%. Furthermore, the experimental data (548 tests) used by the Narasimha model resulted in a $\varepsilon_{Narasimha}$ of 6.98% and $R^2_{Narasimha}$ of 0.938 (shown in Fig. 7), whereas the new model from this research resulted in a $\varepsilon_{New model}$ of 4.53% and $R^2_{New model}$ of 0.973. Nevertheless, the predicted Narasimha model can provide accurate predictions, especially in the range of α_{pred} 2–6. When the value of α_{pred} was higher than 6, the accuracy in the prediction for the Narasimha model gradually decreased.

From Eq. (24), it can be inferred that the decrease in the vortex finder diameter and cone angle of the hydrocyclone led to decreasing separation sharpness. In contrast, a decrease in the spigot diameter of the hydrocyclone, in the relative particle density, and in the slurry viscosity resulted in increasing separation sharpness, which agreed with the conclusions of other researchers [19,22]. The slurry viscosity and density were interrelated and the effect of these variables on the separation sharpness cannot be easily decoupled [23]. In addition to these variables, there are other variables affecting the separation sharpness. The increasing P -number, conical length, hydrocyclone inclination, and hindered settling factor led to a higher separation sharpness. On the other hand, a lower the inlet diameter, Reynolds number, vortex finder length and cylindrical length led to a higher separation sharpness.

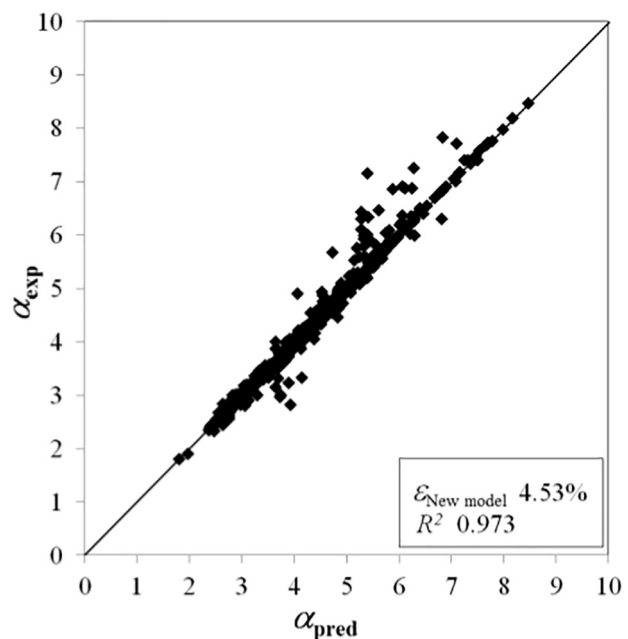


Fig. 6. Comparison of α_{exp} and α_{pred} of the new model.

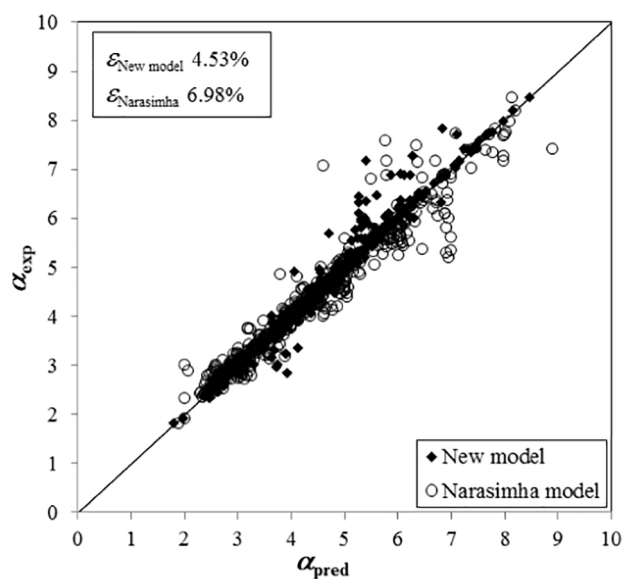


Fig. 7. Comparison of the new model and Narasimha (2014) model.

As described above, the new model has many advantages in improving the prediction of separation sharpness including the following:

- 1) The influence of dimension variables (*i.e.* inlet diameter, vortex finder diameter, spigot diameter, cone angle, vortex finder, cylindrical and conical lengths), feed materials, and medium variables (*i.e.* viscosities of slurry and liquid, densities of slurry and solids, and the hindered settling factor), and operating variables (*i.e.* the hydrocyclone inclination, Reynolds number, and P-number) on separation sharpness were included in the model.
- 2) Significant improvement in separation sharpness prediction.
- 3) The entire data-sets used to develop the model had a wide range of feed solid concentrations from 0.5wt%–80 wt%.
- 4) The Reynolds number and P-number were used to investigate the flow mechanism in the hydrocyclone and the viscosity and hindered settling factor, which were used to consider the effect of solid concentrations on the hydrocyclone performance, leading to improved model prediction.5) The new model considered the effect of the fine fraction (below 38 μm) on slurry viscosity, which covered both Newtonian and non-Newtonian fluids. In other words, this new model can be applied to submicron feed materials covering a wide range of slurry types.

5. Conclusions

The new model was developed from a database consisting of the entire historical data-sets of 431 tests and additional experiments of 117 tests, which were conducted by the authors to fill the gaps in the historical data-sets (*e.g.* the small hydrocyclone body diameter, low feed solid concentration, and various types of feed materials). Furthermore, the dimensionless approach was used to formulate a new model.

In the analysis of the relationship of the dimension, feed material, medium, and operating variables with separation sharpness using the multiple linear regression technique, the results showed that the system constant, A , was refitted for the given type of materials and hydrocyclone body diameters and regression coefficients $n_1 = -0.284$, $n_2 = 0.135$, $n_3 = -0.158$, $n_4 = 0.068$, $n_5 = -0.140$, $n_6 = -0.682$, $n_7 = -0.173$, $n_8 = 0.242$, $n_9 = 0.182$, $n_{10} = 0.346$, $n_{11} = -0.088$, $n_{12} = -0.190$ and $n_{13} = 0.329$. Among the regression coefficients of the independent variables that influenced the separation sharpness, the high value of the exponent for

these coefficients of the vortex finder length, l/D_c , hindered settling factor, $(1 - f_v)^{2/10^{(1.82f_v)}}$, cone angle, $\tan(\theta/2)$, feed inlet diameter, D_i/D_c , conical length, L_{co}/D_c , relative slurry viscosity, μ_s/μ_l , hydrocyclone inclination, $\cos(i/2)$, cylindrical length, L_c/D_c , spigot diameter, D_u/D_c , Reynolds number, Re , and vortex finder diameter, D_o/D_c , had a higher effect than the coefficients of the relative particle density, $(\rho_s - \rho_s)/\rho_s$, and the P-number, γ . The new separation sharpness model can successfully be used to predict experimental separation sharpness. When compared to the Narasimha model, the accuracy in prediction of the new model was significantly improved.

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